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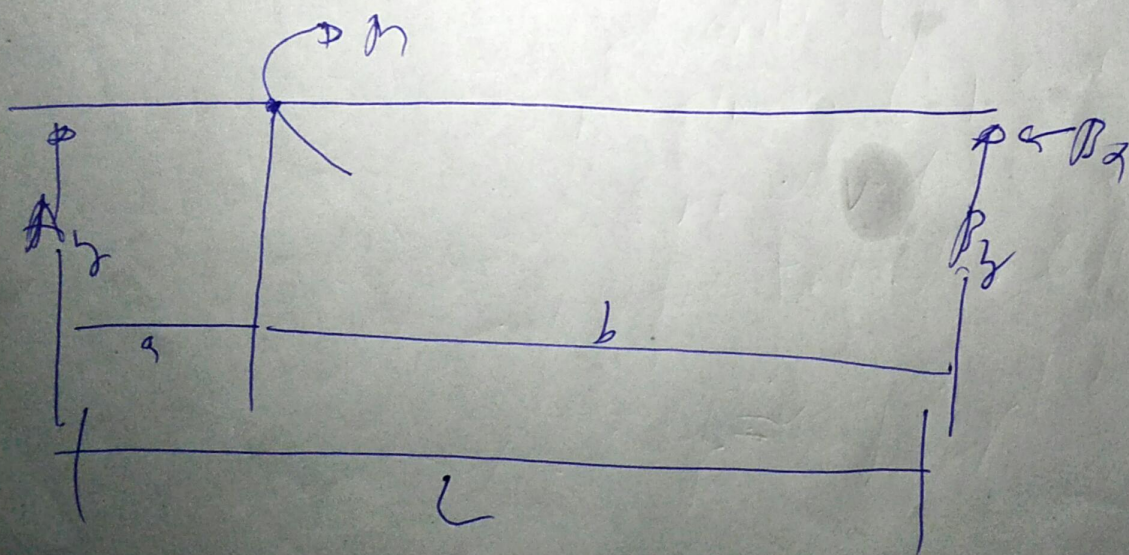
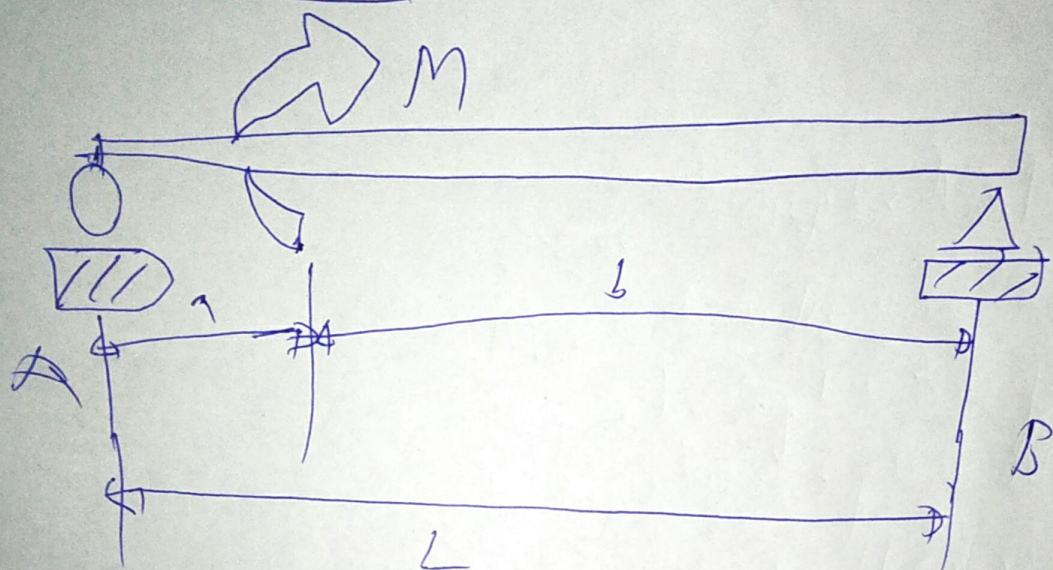
LBX LV2F

Computational Exercise: CompEx No. 1 -

Shear and Bending Moment in Beams

1. Draw the shear and bending moment diagrams for each given beam. Show complete solution for solving reactions as well as shear and moment values at critical points.
 2. Determine the maximum (+) and minimum (-) shear and moment values and indicate their location from the leftmost point.
- Note: Neglect the self-weight of beam.

Problem No. 1



Solution!

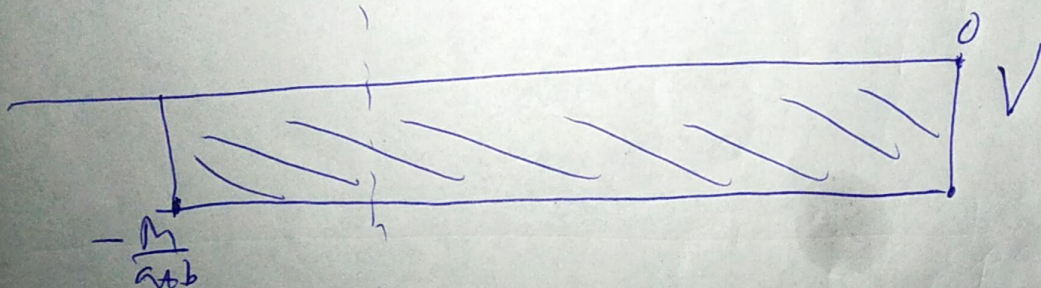
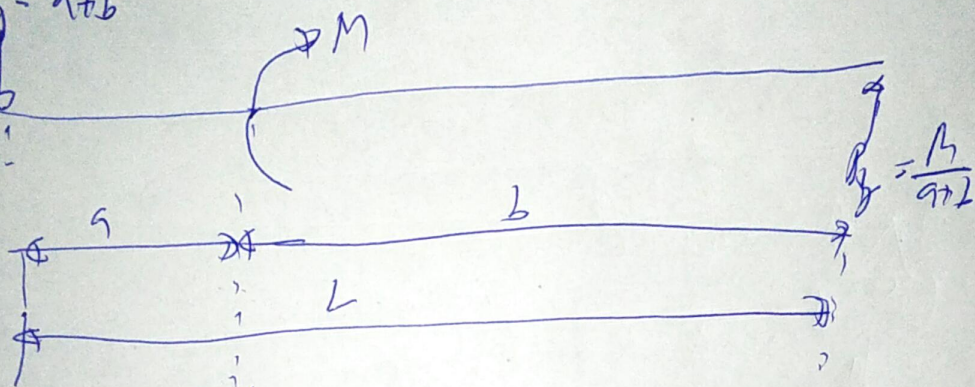
$$\sum F_x = 0: B_x = 0$$

$$\sum M_A = 0: \odot M - B_y (a+b) = 0$$

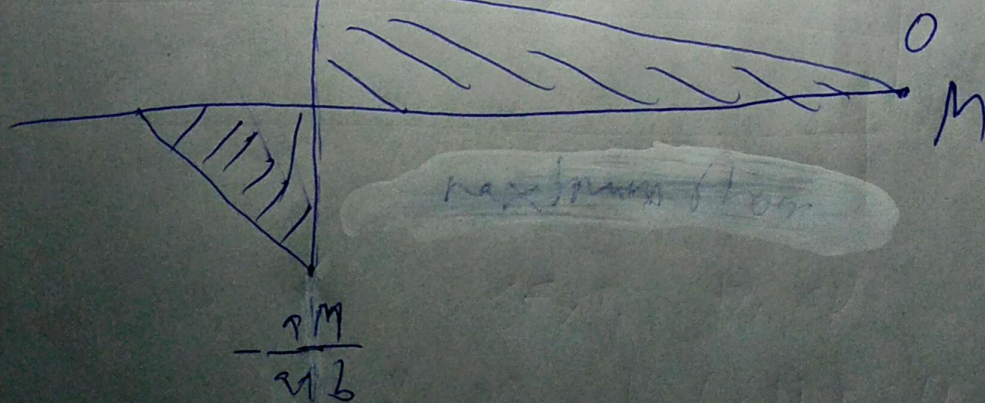
$$B_y = \frac{M}{a+b}$$

$$\sum F_y = 0: A_y + B_y = 0 \Rightarrow A_y + \frac{M}{a+b} = 0 \Rightarrow A_y = -\frac{M}{a+b}$$

$$A_y = \frac{M}{a+b}$$

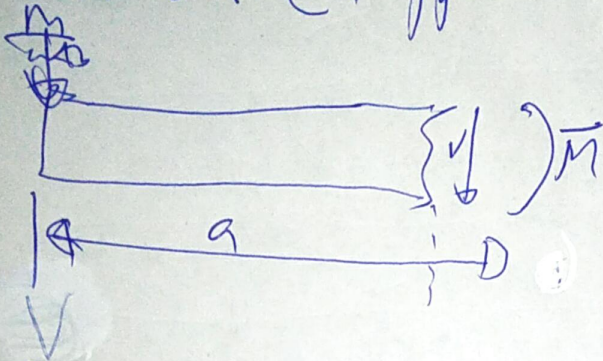


$$-\frac{aM}{a+b} + M \frac{(a+b)}{a+b} = \frac{bM}{a+b}$$



maximum stress

as $x \rightarrow a^-$ (x approaches a from the left hand side):



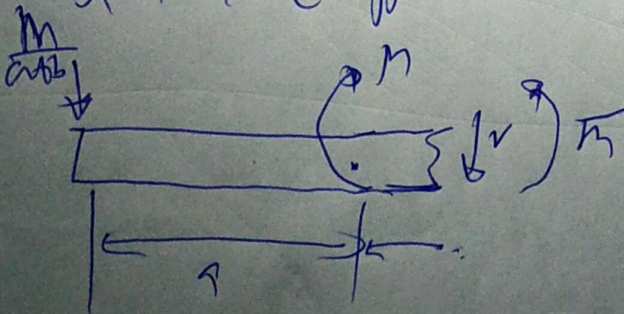
$$\sum F_y = 0:$$

$$-\frac{M}{a+b} - V = 0 \Rightarrow V = -\frac{M}{a+b}$$

$$(\sum M = 0: +)$$

$$\frac{M}{a+b} (a) + \bar{M} = 0 \Rightarrow \bar{M} = -\frac{aM}{a+b}$$

$x \rightarrow a^+$ (x approaches a from the right hand side):



$$\sum F_y = 0:$$

$$-\frac{M}{a+b} - V = 0 \Rightarrow V = -\frac{M}{a+b}$$

$$(\sum M = 0: +)$$

$$\frac{M}{a+b} - M + \bar{M} = 0 \Rightarrow \bar{M} = -\frac{M}{a+b} + M = -\frac{M}{a+b} + M \left(\frac{a+b}{a+b} \right)$$

$$\bar{M} = \frac{bM}{a+b}$$

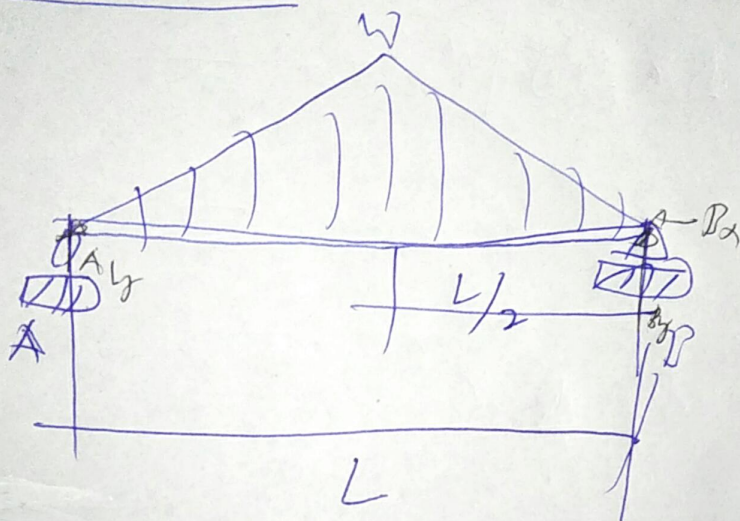
$$\max_{x \in [0, a+b]} V = 0 \text{ at } x=0 \text{ and } x=a+b$$

$$\min_{x \in [0, a+b]} V = \frac{17}{a+b} \text{ at } x \in \{x \mid 0 < x < a+b\}$$

$$M_{\max} = \frac{bM}{a+b} \text{ at } x=s$$

$$M_{\min} = -\frac{aM}{a+b} \text{ at } x=0$$

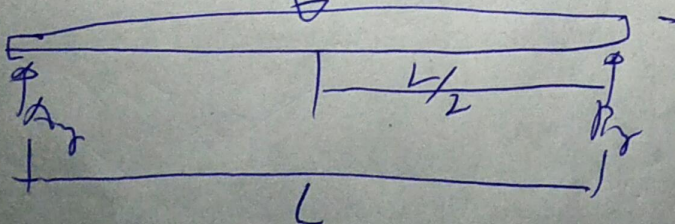
Problem No. 2



$\sum F_x = 0$

$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum F_y = 0 \Rightarrow A_y + B_y = \frac{wL}{2}$$

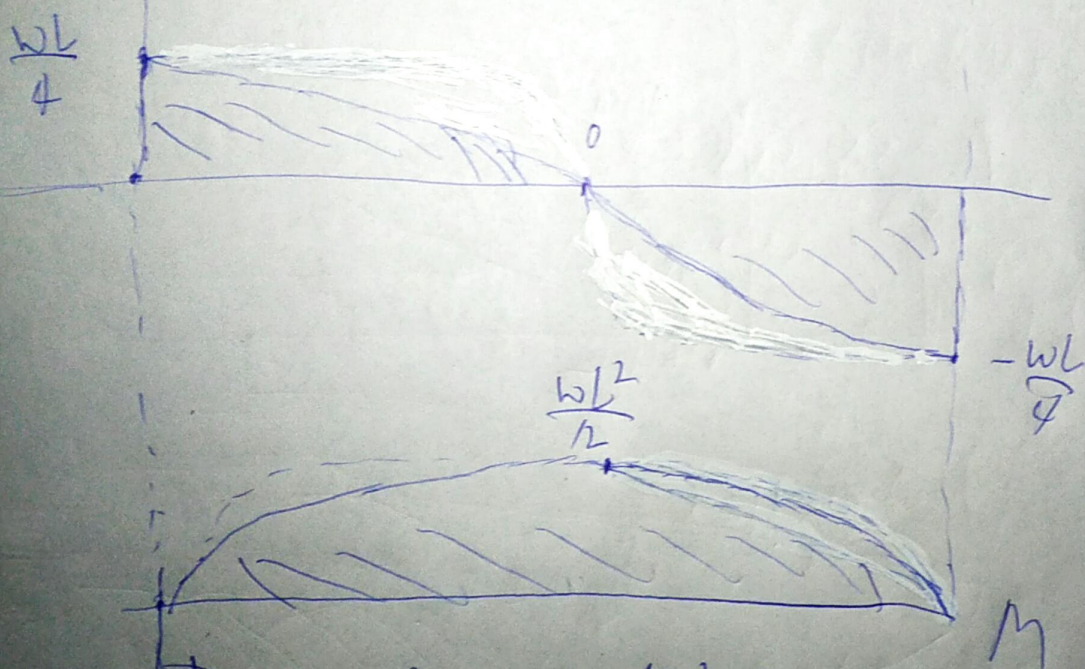
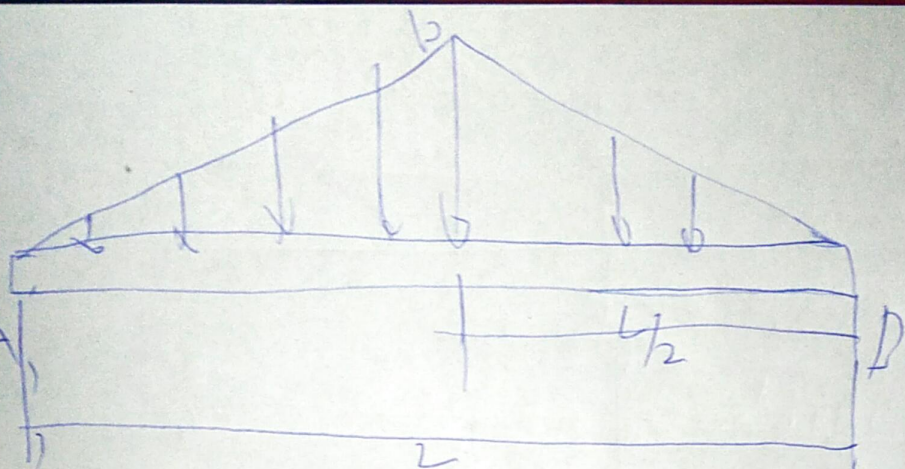


$$\sum M_D = 0 \quad (+)$$

$$w \left(\frac{L}{2} \right) \left(\frac{L}{2} \right) - A_y (L) = 0 \Rightarrow A_y = \frac{wL}{4}$$

$$\sum F_y = 0$$

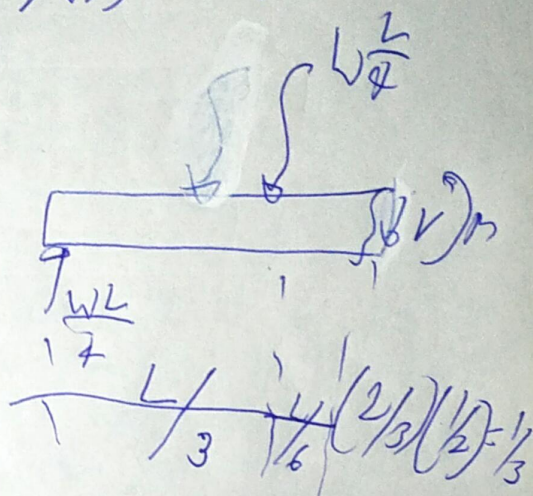
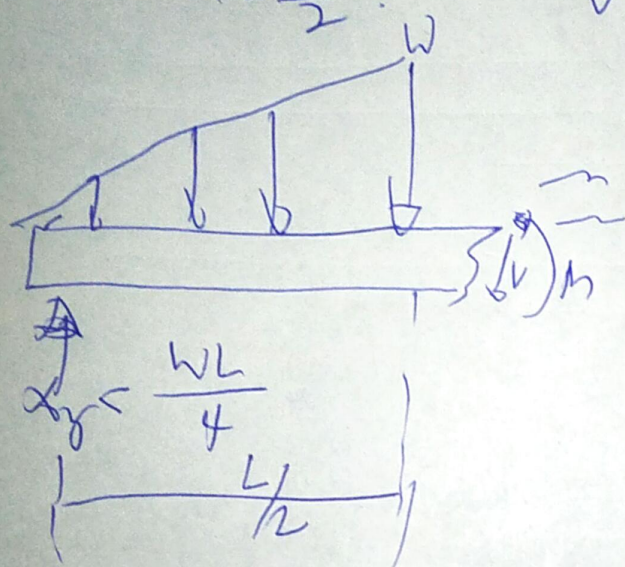
$$\frac{wL}{4} - \frac{wL}{2} + B_y = 0 \Rightarrow B_y = \frac{wL}{4}$$



Increasing σ over, flat σ , steep
Decreasing σ , steep then flat

$$\text{so } \alpha = \frac{L}{2}!$$

$$V\left(\frac{L}{2}\right)\left(\frac{L}{2}\right) = V\frac{L}{4}$$



$$\sum F_x = 0!$$

$$\frac{WL}{4} - \frac{WL}{4} - V = 0 \Rightarrow V = 0$$

$$\sum M = 0! (+)$$

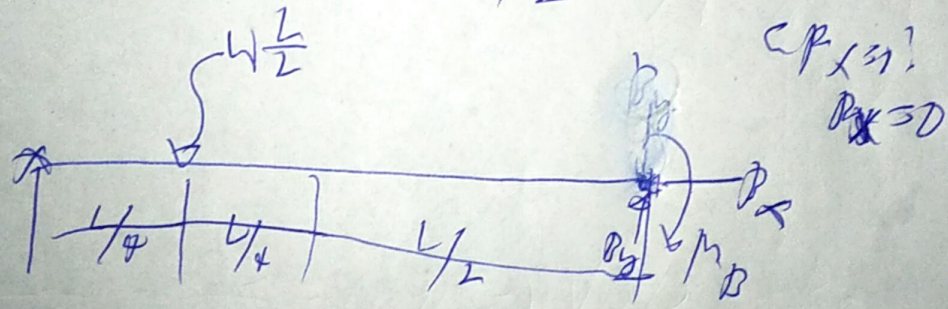
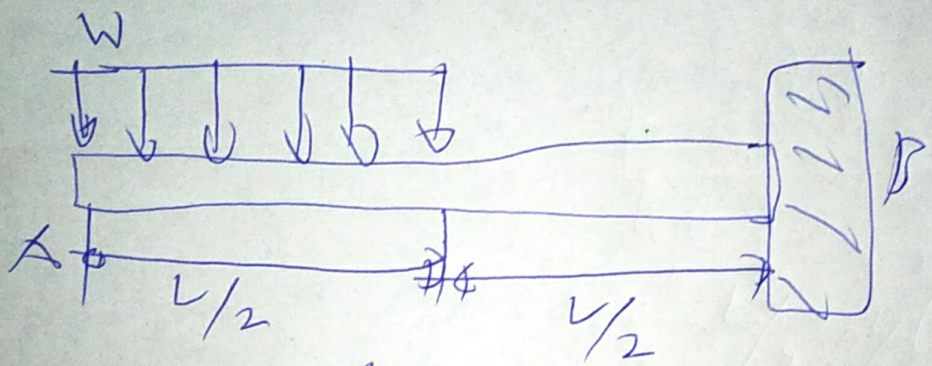
$$-\frac{WL}{4}\left(\frac{L}{2}\right) + \frac{WL}{4}\left(\frac{L}{6}\right) + M = 0$$

$$M = \frac{WL^2}{8} - \frac{WL^2}{24} = \frac{WL^2}{12}$$

$$-\frac{WL}{4}\left(\frac{L}{3}\right) + M = 0 \Rightarrow M = \frac{WL^2}{12}$$

$$\begin{aligned} \max V &= \frac{WL}{4} \text{ at } x=0 & \max M &= \frac{WL^2}{12} \text{ at } x=\frac{L}{2} \\ \min V &= -\frac{WL}{4} \text{ at } x=L & \min M &= 0 \text{ at } x=0, L \end{aligned}$$

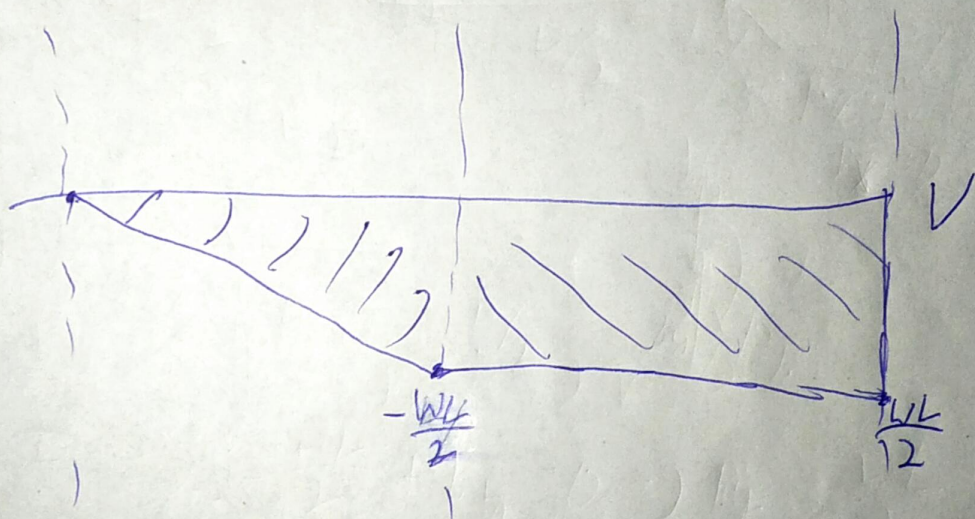
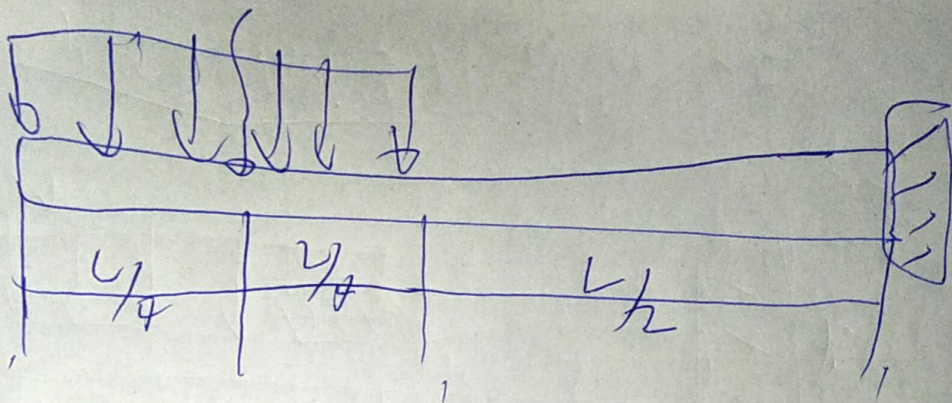
Problem No. 3



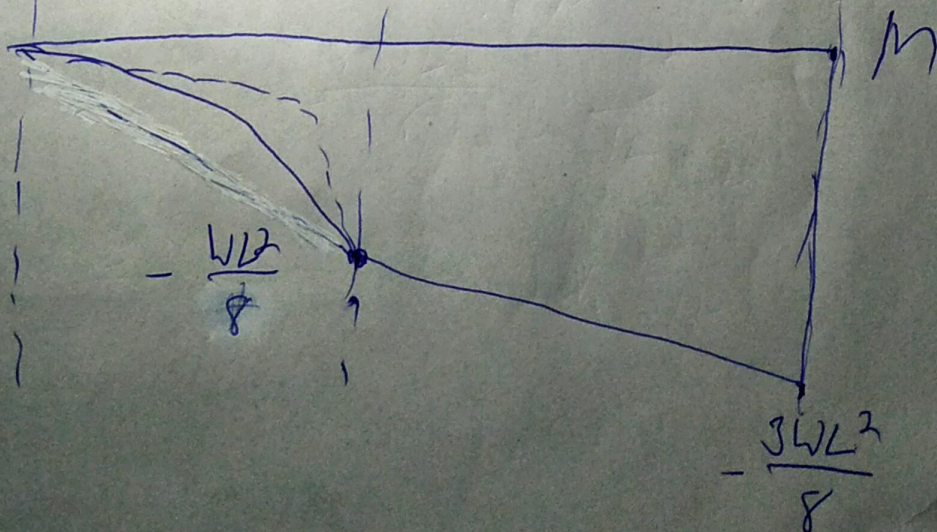
$$\sum P_y = 0 \therefore -\frac{WL}{2} + P_y = 0 \Rightarrow P_y = \frac{WL}{2}$$

$$\sum M_B = 0 \therefore (+) -\frac{WL}{2} \left(\frac{3L}{4} \right) + M_B = 0$$

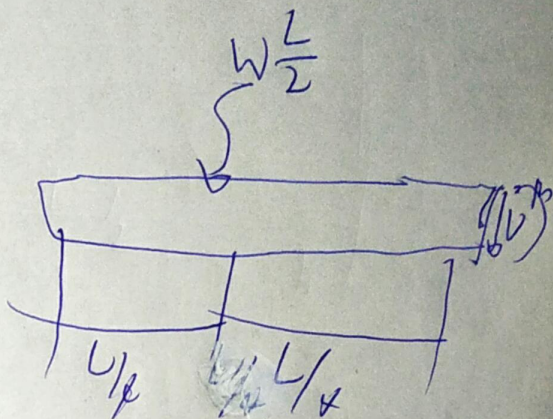
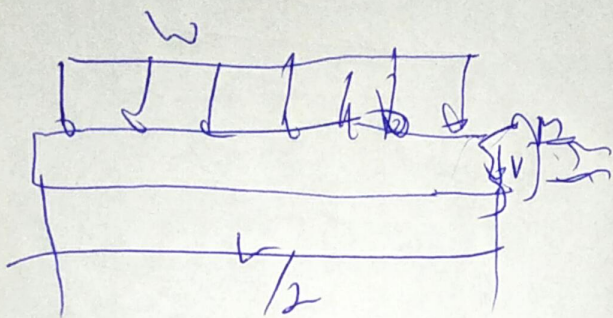
$$M_B = \frac{3WL^2}{8}$$



$$\left(\frac{1}{2}\right) \frac{wL}{2} \left(\frac{L}{2}\right) = \frac{wL^2}{8}$$



$$x = L/2:$$



$$F_y = 0:$$

$$-\frac{WL}{2} - V = 0 \Rightarrow V = -\frac{WL}{2}$$

$$M = 0:$$

$$W \frac{L}{2} \left(\frac{L}{4} \right) + M = 0 \Rightarrow M = -\frac{WL^2}{8}$$

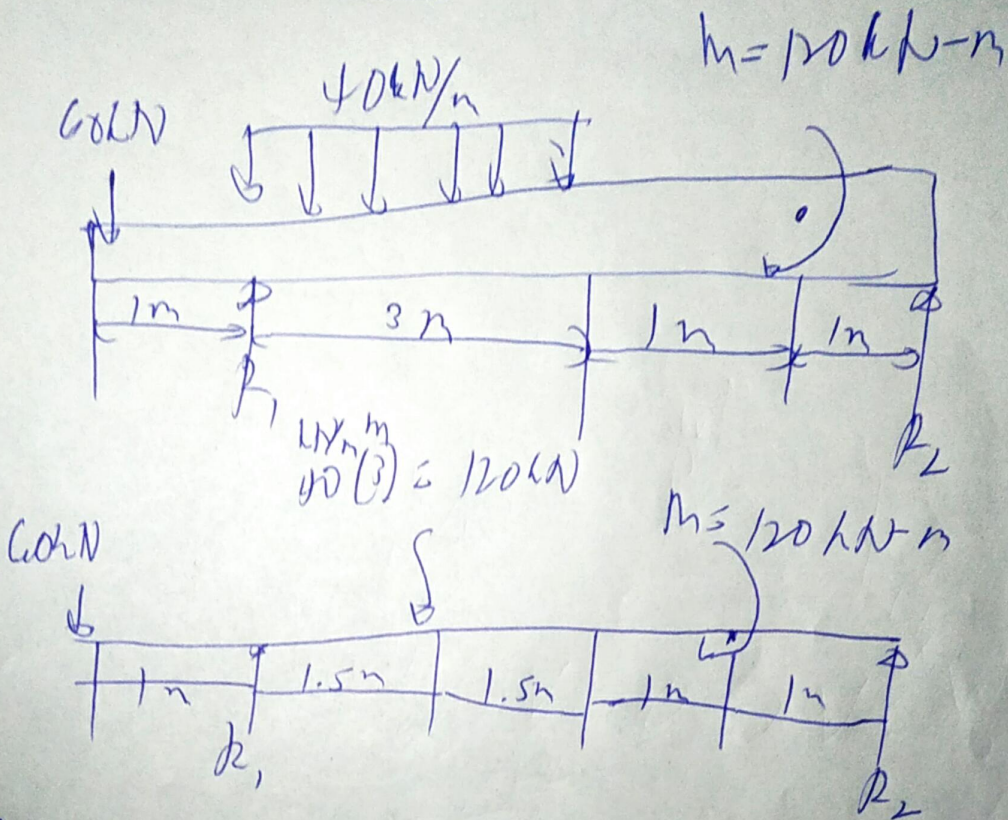
$$\max V = 0 \text{ at } x = 0, L$$

$$\min V = -\frac{WL}{2} \text{ at } x = [L/2, L)$$

$$\max M = 0 \text{ at } x = 0, L$$

$$\min M = -\frac{3WL}{8} \text{ at } x = L$$

Problem No. 4



$$M_{R_1} = 0 \quad + \curvearrowright$$

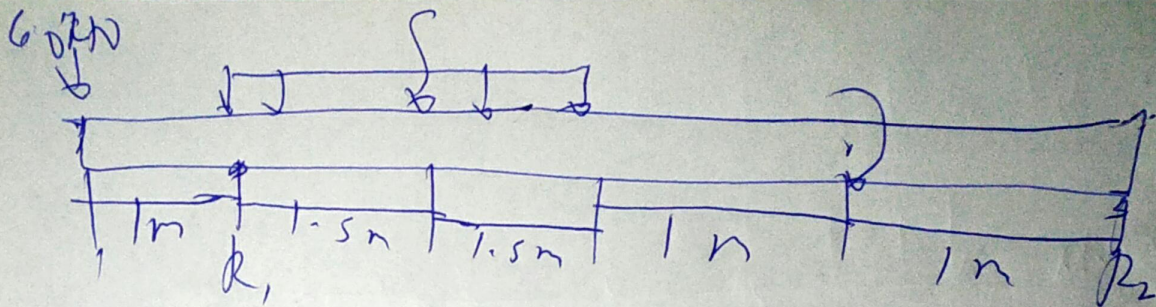
$$-60 \text{ kN}(1\text{m}) + 120 \text{ kN}(1.5\text{m}) + 120 \text{ kN-m} - R_2(5\text{m}) = 0$$

$$R_2 = 48 \text{ kN}$$

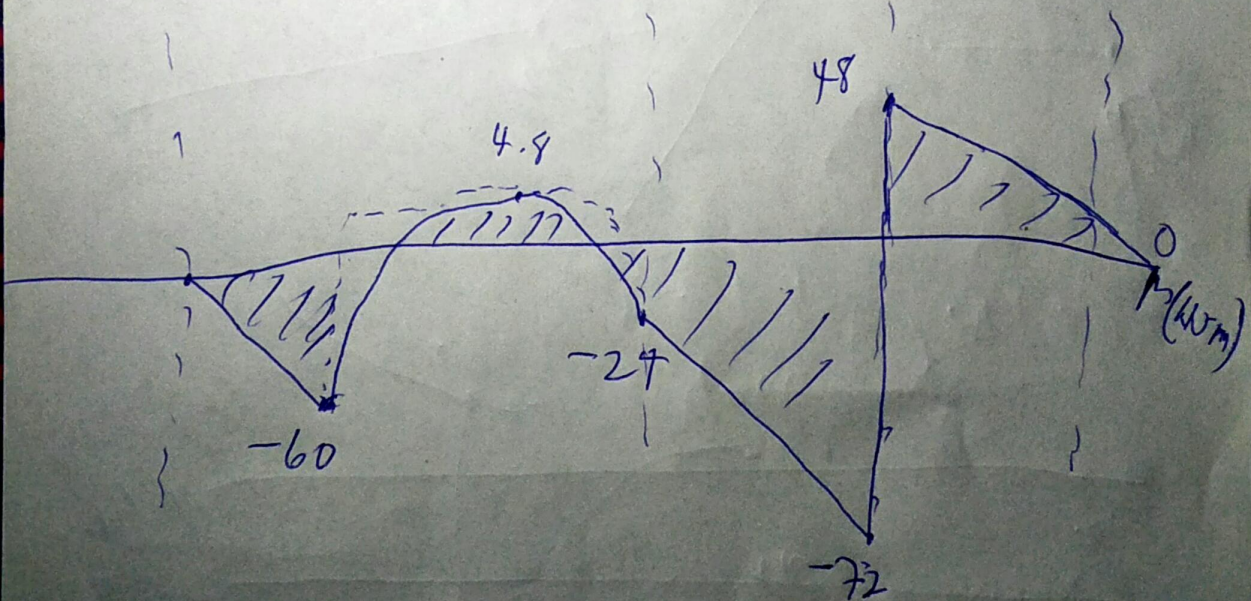
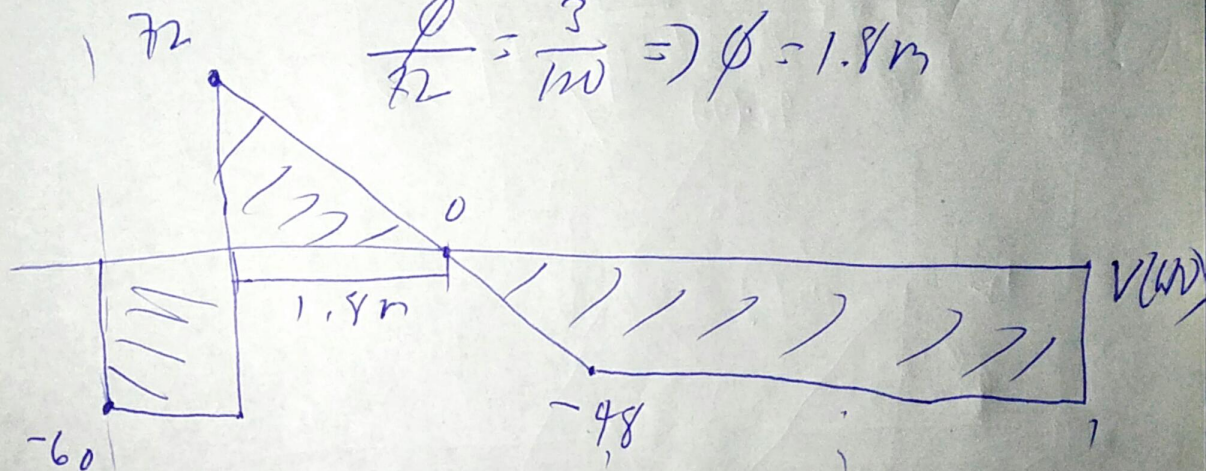
$$\sum R_y = 0$$

$$-60 \text{ kN} + R_1 - 120 \text{ kN} + 48 \text{ kN} = 0$$

$$R_1 = 132 \text{ kN}$$

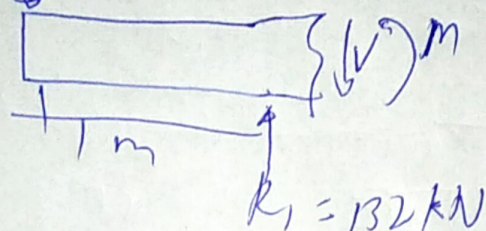


$$\frac{\phi}{72} = \frac{3}{120} \Rightarrow \phi = 1.8\text{m}$$



$$\alpha x = 1 \text{ m}$$

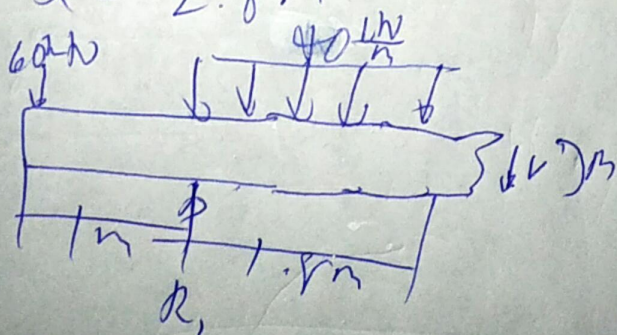
60 kN



$$\sum F_y = 0: -60 \text{ kN} + \overset{132 \text{ kN}}{R_1} - V = 0 \Rightarrow V = 72 \text{ kN}$$

$$\sum M = 0 (+): 60 \text{ kN} (1 \text{ m}) + M = 0 \Rightarrow M = -60 \text{ kNm}$$

$$\alpha x = 2.8 \text{ m}$$

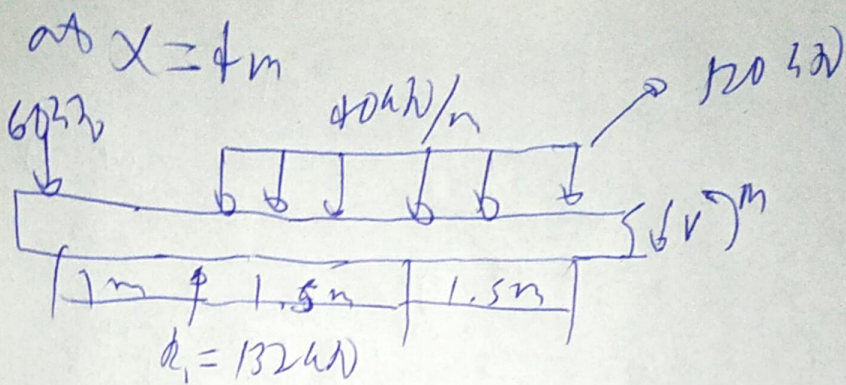


$$Q = \frac{40 \text{ kN}}{\text{m}} (1.8 \text{ m}) = 72 \text{ kN}$$

$$\sum F_y = 0: -60 \text{ kN} + 132 \text{ kN} - 72 \text{ kN} - V = 0 \Rightarrow V = 0$$

$$\sum M = 0 (+): 60(2.8) - 132(1.8) + 72(0.9) + M = 0$$

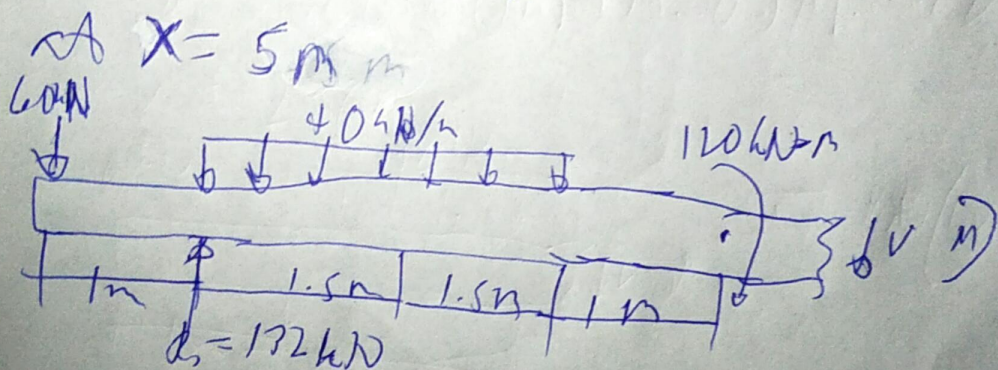
$$M = 4.8 \text{ kNm}$$



$$\sum F_y = 0: -60 + 132 - 120 - V = 0 \Rightarrow V = -48\text{ kN}$$

$$\sum M = 0 (\circlearrowleft): -60(4) + 132(3) - 120(1.5) - M = 0$$

$$M = -24\text{ kN-m}$$



$$\sum F_y = 0: -60 + 132 - 120 - V = 0 \Rightarrow V = -48\text{ kN}$$

$$\sum M = 0 (\circlearrowleft): -60(5) + 132(4) - 120(2.5) + 120 - M = 0$$

$$M = 48\text{ kN-m}$$

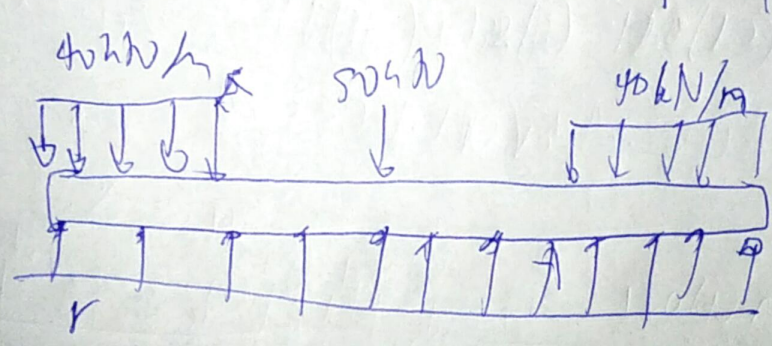
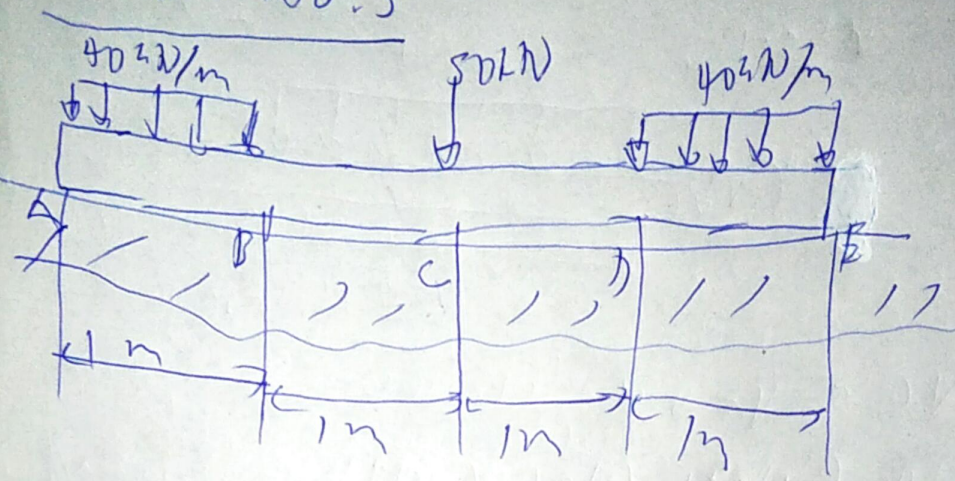
$$\text{max } V = 72\text{ kN at } x < 1\text{ m}$$

$$\text{min } V = -48\text{ kN at } x = 4\text{ m}$$

$$\text{max } M = 48\text{ kN-m at } x = 5\text{ m}$$

$$\text{min } M = -42\text{ kN-m at } x = 5\text{ m}$$

Problem No. 5

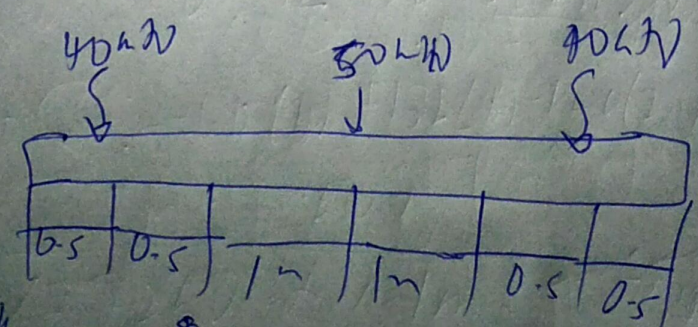


$$R = 40(4)$$

$$R = \frac{R}{4m}$$

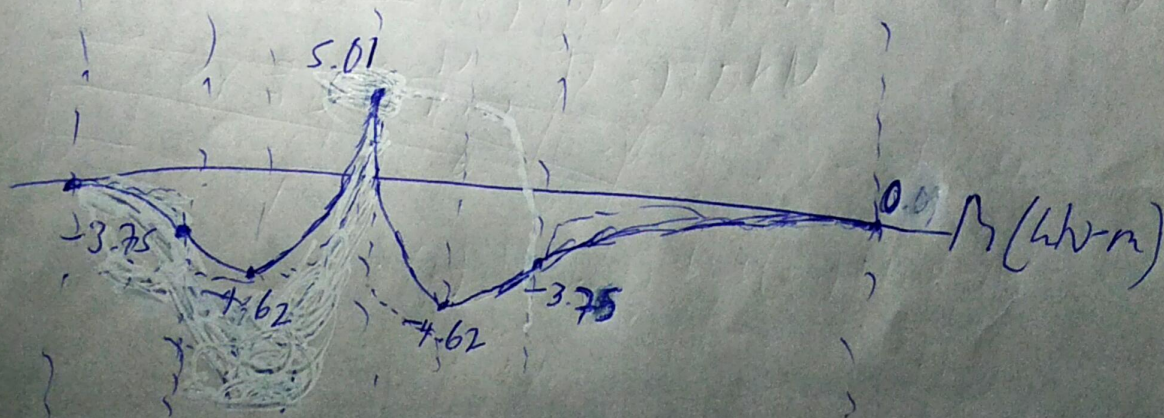
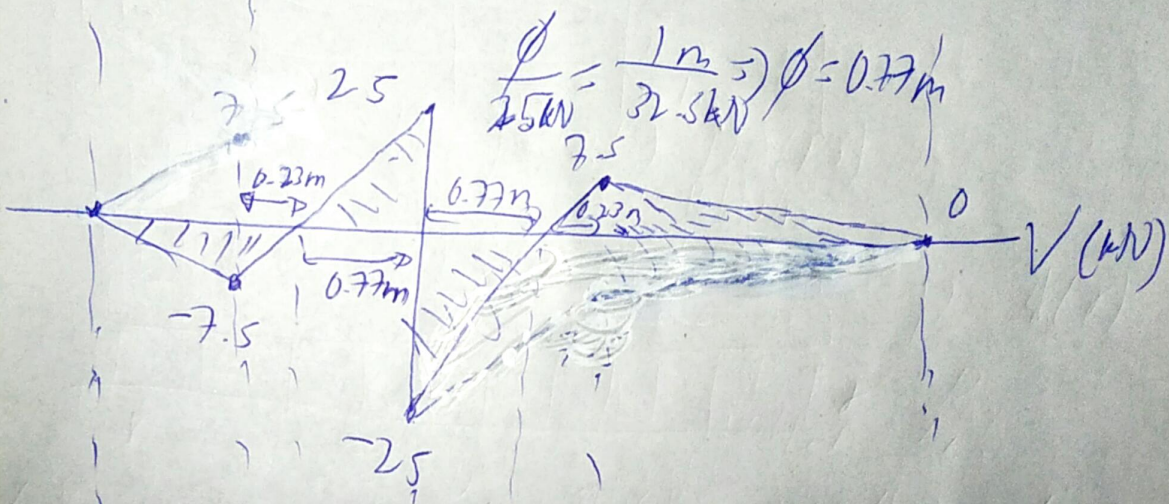
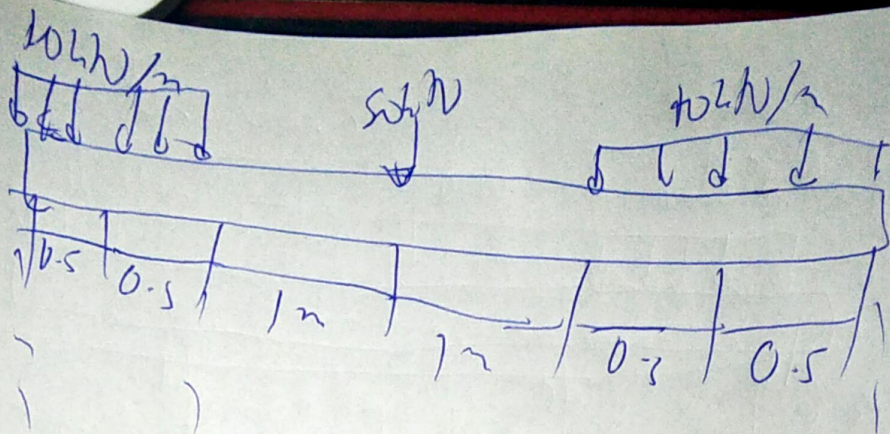
$$\sum F_y = 0: -40 \frac{kN}{m}(1m) - 50kN - 40 \frac{kN}{m}(1m) + R(4m) = 0$$

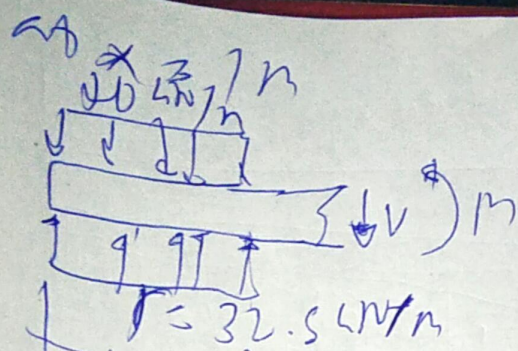
$$R = 32.5 \frac{kN}{m}; R = 130 kN$$



$$\sum M_A = 0: 40(0.5) - 50(2) + R(2) - 40(0.5) - 14 = 0$$

$$R = 0 kN-m$$

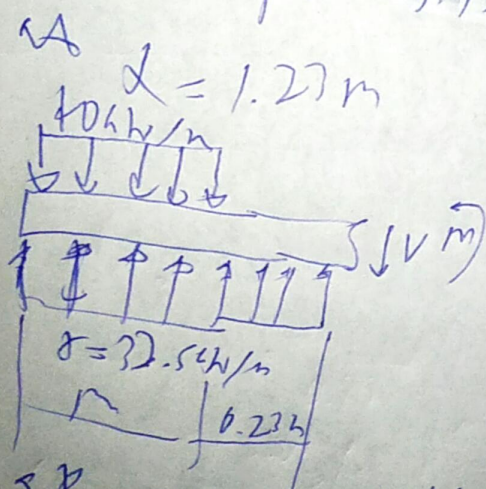




$$\sum F_y = 0: -40 \text{ kN/m}(0.5\text{m}) + 32.5(0.5\text{m}) - V = 0 \Rightarrow V = 7.5 \text{ kN}$$

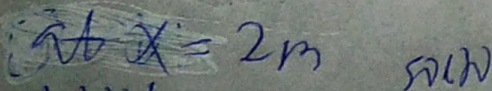
$$\sum M = 0: -40 \text{ kN}(0.5\text{m}) + 32.5(0.5\text{m}) - M = 0$$

$$M = -3.75 \text{ kNm}$$



$$\sum F_y = 0: -40 \text{ kN/m}(0.62\text{m}) + 32.5(0.62) - V = 0 \Rightarrow V = 0 \text{ kN}$$

$$\sum M = 0: -40 \text{ kN}(0.48\text{m}) + 32.5(0.62) - M = 0 \Rightarrow M = -4.62 \text{ kNm}$$

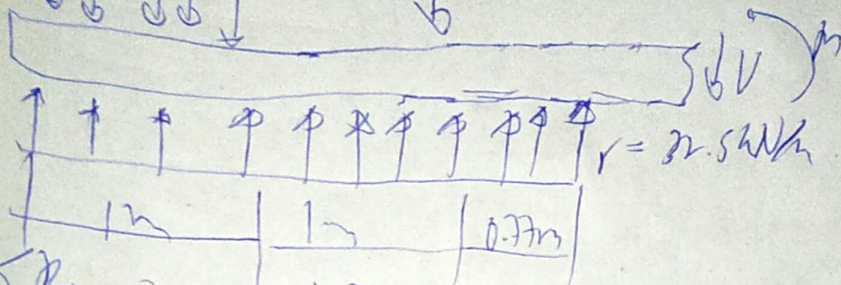
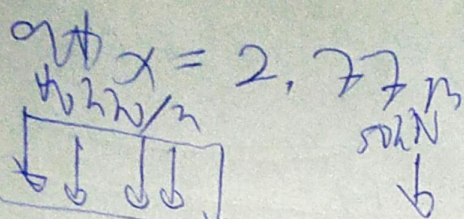


$$\sum F_y = 0$$

$$-40 \text{ kN/m}(1\text{m}) + 32.5(2) - V = 0$$

$$V = 25 \text{ kN}$$

$$\sum M = 0: -40 \text{ kN}(1.5\text{m}) + 32.5(1) - M = 0 \Rightarrow M = 5.0 \text{ kNm}$$

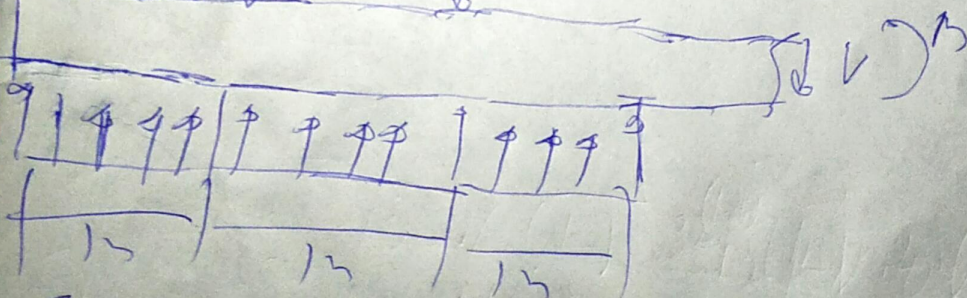
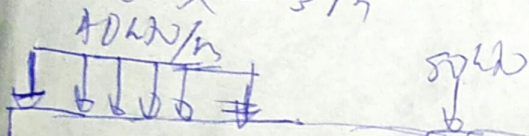


$$\sum F_y = 0: -40(1) - 50 + 32.5(2.77) - V = 0 \Rightarrow V = 0$$

$$\sum M = 0 (+): -40(2.27) - 50(0.77) + 32.5(2.77)(1.39) - M = 0$$

$$M = -4.62\text{ kN-m}$$

at $x = 3\text{ m}$



$$\sum F_y = 0: -40(1) - 50 + 32.5(3) - V = 0 \Rightarrow V = 25\text{ kN}$$

$$\sum M = 0 (+): -40(2.5) - 50(1) + 32.5(3)(1.5) - M = 0$$

$$M = -3.75\text{ kN-m}$$

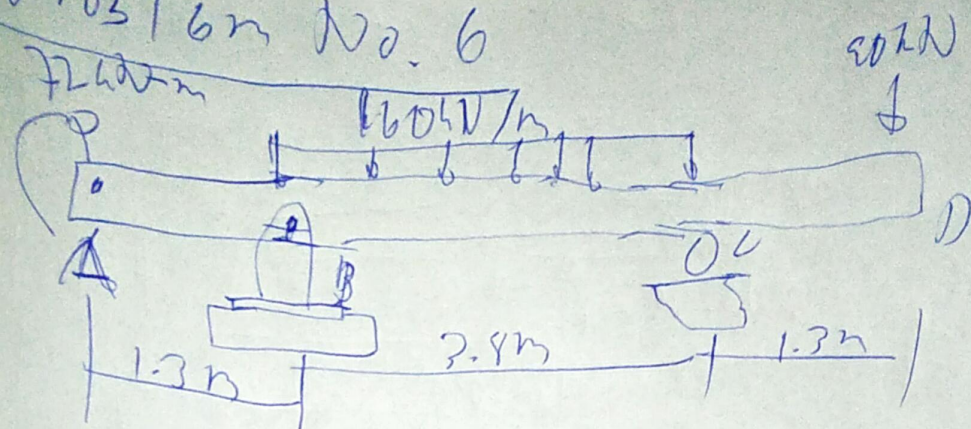
max $V = 25\text{ kN}$ at $x = 3\text{ m}$

min $V = -25\text{ kN}$ at $x = 2\text{ m}$

max $M = 5.01\text{ kN-m}$ at $x = 2\text{ m}$

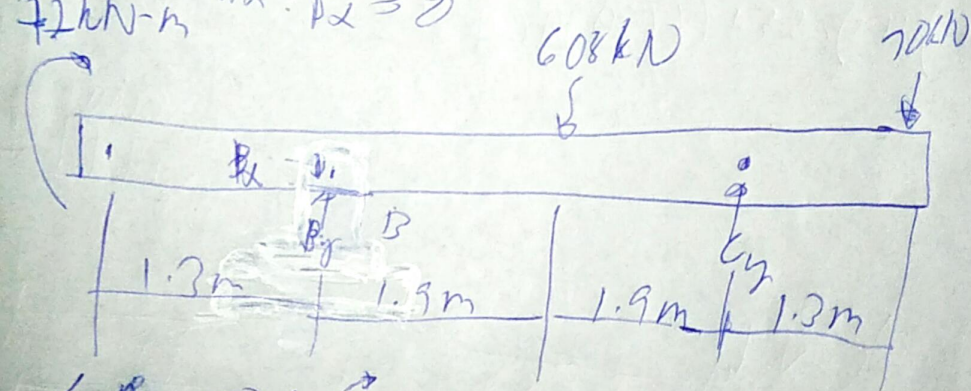
min $M = -4.62\text{ kN-m}$ at $x = 1.23\text{ m}, 2.77\text{ m}$

Prob 16m No. 6



$$W = 160 \text{ kN/m} (3.8 \text{ m}) = 608 \text{ kN}$$

$$\sum F_x = 0: B_x = 0$$



$$\sum M_D = 0: (+)$$

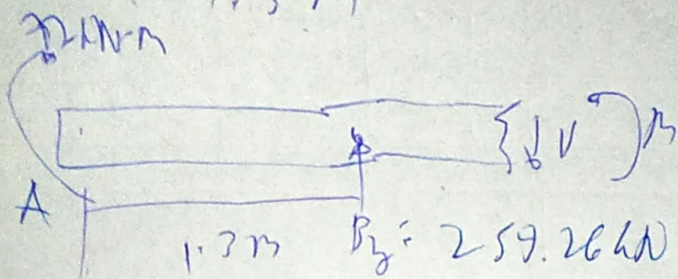
$$72 \text{ kN-m} + 608 \text{ kN} (1.9 \text{ m}) - C_y (3.8 \text{ m}) + 90 \text{ kN} (5.1 \text{ m}) = 0$$

$$C_y = 443.74 \text{ kN}$$

$$\sum F_y = 0: B_y - 608 \text{ kN} + 443.74 \text{ kN} - 90 \text{ kN} = 0$$

$$B_y = 254.26 \text{ kN}$$

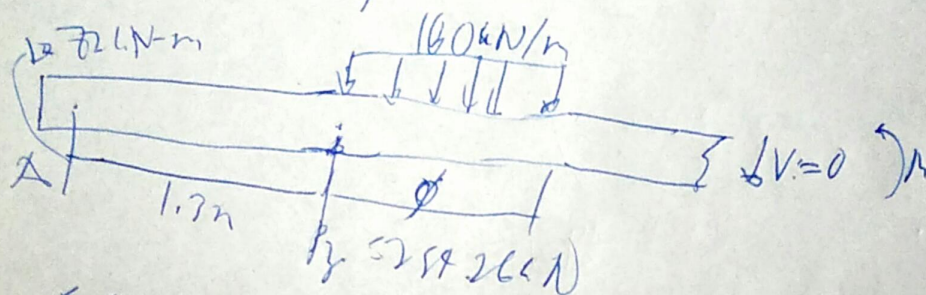
$$x = 1.3 \text{ m}$$



$$\sum F_y = 0: 254.26 \text{ kN} - V = 0 \Rightarrow V = 254.26 \text{ kN}$$

$$\sum M_A = 0 (+): 72 \text{ kN-m} - 254.26(1.3) - M = 0 \Rightarrow M = -258.538 \text{ kN-m}$$

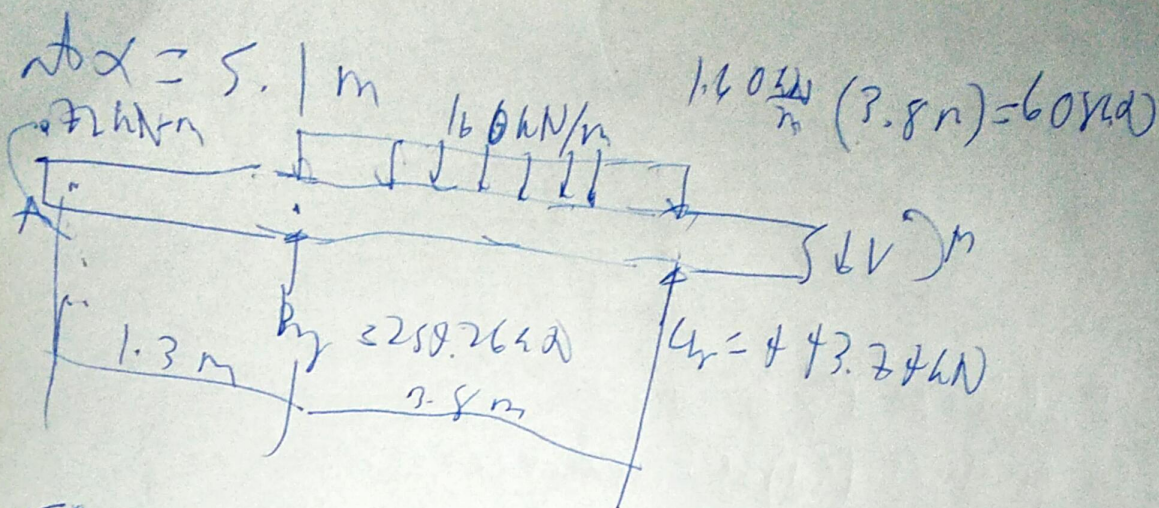
$$x = 1.3 \text{ m} \quad \phi = 2.8852 \text{ m}$$



$$\sum F_y = 0: 254.26 \text{ kN} - 160 \frac{\text{kN}}{\text{m}} \phi = 0 \Rightarrow \phi = 1.5892 \text{ m}$$

$$\sum M_A = 0 (+): 72 - 254.26(1.3) + 160 \phi \left(\frac{1.3 + \phi}{2} \right) - M = 0$$

$$M = 278.0601 \text{ kN-m}$$



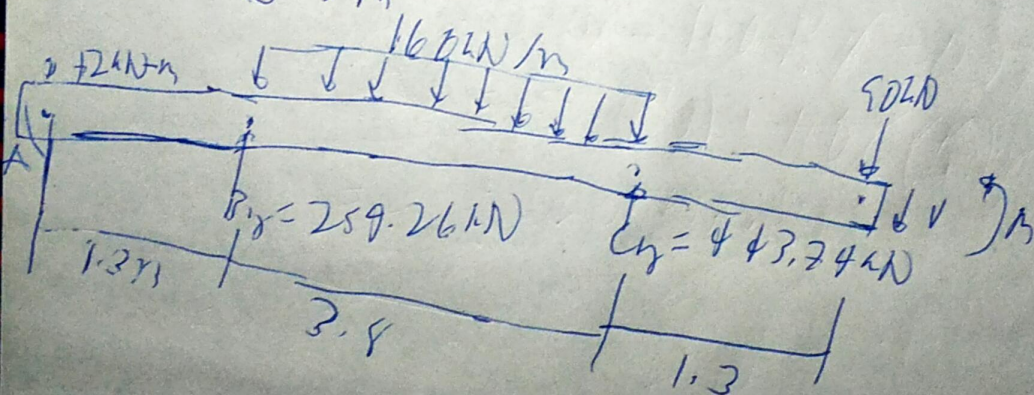
$$\sum F_y = 0: 259.26 \text{ kN} - 608 \text{ kN} + 443.74 \text{ kN} - V = 0$$

$$V = 90 \text{ kN}$$

$$\sum M_A = 0: 72 - 259.26(1.3) + 608(3.8) - 443.74(5.1) - M = 0$$

$$M = -577.7842 \text{ kN-m}$$

$$x = 6.4 \text{ m}$$

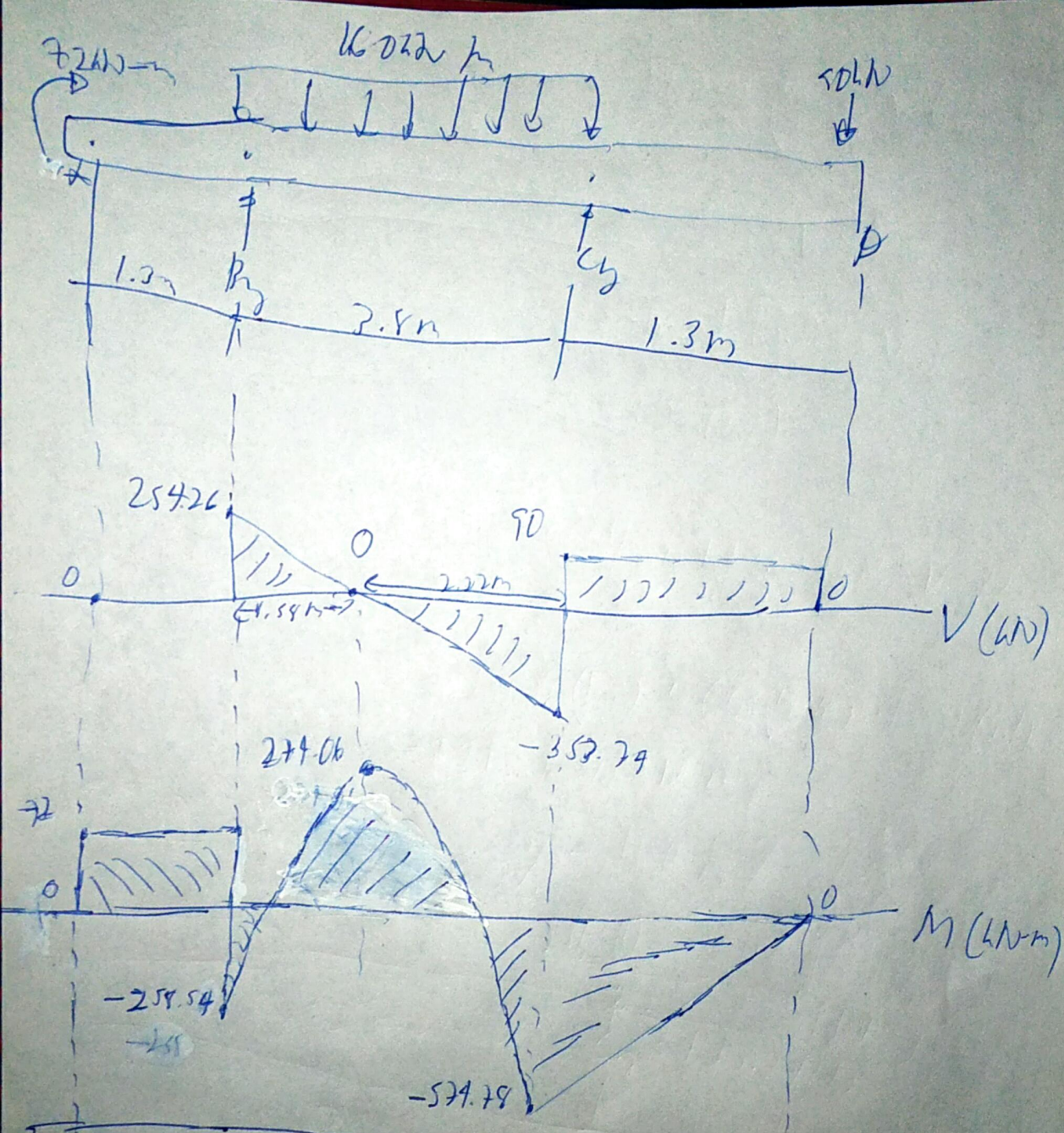


$$\sum F_y = 0: 259.26 \text{ kN} - 608 \text{ kN} + 443.74 \text{ kN} - 50 \text{ kN} - V = 0$$

$$V = 0 \text{ kN}$$

$$\sum M_x = 0: 72 - 259.26 \text{ kN}(1.3 \text{ m}) + 160 \text{ kN/m} (3.8 \text{ m})(1.3 \text{ m}) - 443.74 \text{ kN}(5.1 \text{ m}) + 50 \text{ kN}(6.4 \text{ m}) - M = 0$$

$$M = -0.012 \text{ kN-m} \approx 0 \text{ kN-m}$$



$\max V = 254.26 \text{ kN} \quad \text{at } x = 1.3 \text{ m}$
 $\min V = -353.79 \text{ kN} \quad \text{at } x = 5.1 \text{ m}$
 $\max M = 274.06 \text{ kN-m} \quad \text{at } x = 2.85 \text{ m}$
 $\min M = -574.78 \text{ kN-m} \quad \text{at } x = 5.1 \text{ m}$